## Lecture 2 «Convective heat exchange. Equation of heat emission. Criteria of thermal similarity»

**Aim:** Formulate the convective heat exchange. Write the equation of heat emission. Describe the criteria of thermal similarity.

**Lecture summary:** Heat exchange between a surface of a solid body and the liquid or gaseous environment in case of their direct contact is called as an emission (or a thermolysis) or convective heat exchange. In case of convective heat exchange transfer is warm from a surface of a solid body in a core of the liquid environment or from the liquid environment to a surface of a solid body is performed by heat conductivity and convection.

Intensity of convective heat exchange is generally determined by availability and thickness of a laminar interface of $\delta_{l.i}$. Heat is transmitted through this layer only by heat conductivity.

Thickness of a laminar interface of $\delta_{l.i.}$ depends on a mode of movement of liquid. It decreases with increase in speed of movement of liquid and viscosity reduction. Therefore, intensity of an emission is in direct dependence on the speed of a flow and in the return – from viscosity of the environment.

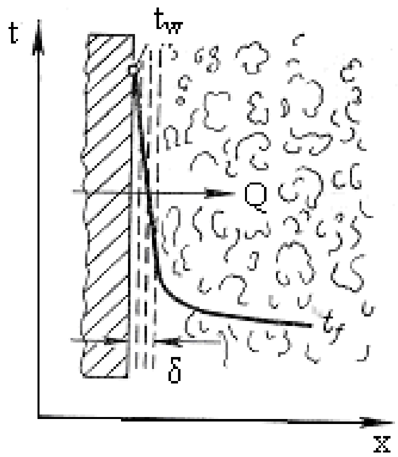


**Fig. 1.** The structure of the thermal and hydrodynamic boundary layers

### Heat emission equation

At the heart of calculations of convective heat exchange Newton's law lies. Quantity of the warmth transferred from a heat exchange surface to the environment surrounding it or, on the contrary, from environment to a heat exchange surface, in direct ratio surface areas of heat exchange of dF, a difference of temperatures between a surface of a body and the environment ($t_s$-$t_{env}$) and time $d\tau$.

$$Q = \alpha F(t_w - t_l)\ \text{, J/kcal} \tag{1}$$

The coefficient of a thermolysis of $\alpha$, characterizes intensity of heat exchange between a surface of a body and environment.

The physical meaning of coefficient $\alpha$ is that it represents quantity of warmth of Q given by unit of a surface in unit of time at a difference of temperatures between a solid surface and environment to one degree. Dimension of coefficient of an emission is from the equation (2)

$$[\alpha] = \left[\frac{Q}{F(t_w - t_l)}\right] = \left[\frac{J}{m^2 \cdot s \cdot K}\right] = \left[\frac{W}{m^2 \cdot K}\right] \tag{2}$$

The coefficient α depends on the physical nature of process, physical properties of substances participating in heat exchange, geometrical characteristics of the equipment and conditions on borders of system in which this process proceeds.

Due to the complex structure of the flows, especially under conditions of turbulent motion, the quantity $\alpha$ is a complex function of many variables.

The heat transfer coefficient depends on the following factors:

- the fluid velocity $w$, its density $\rho$ and the viscosity $\mu$, i.e. variables that determine the regime of fluid flow;

- the thermal properties of the fluid (specific heat capacity $c_p$, thermal conductivity $\lambda$), as well as the coefficient of volumetric expansion $\beta$;

- geometric parameters – the shape and defining dimensions of the wall (for pipes – their diameter $d$ and length $L$), as well as the roughness $\varepsilon$ of the wall.

Thus

$$\alpha = f(w, \mu, \rho, c_p, \lambda, \beta, d, L, \varepsilon) \tag{3}$$

Due to the complex dependence of the heat emission coefficient on a large number of factors, it is impossible to obtain a calculated equation for $\alpha$, which is suitable for all cases of heat emission. Only by generalizing the experimental data with the help of the theory of similarity we can obtain a generalized (criterial) equation for the typical cases of heat emission, which makes it possible to calculate $\alpha$ for the conditions of a specific task.

A mathematical description of the process of heat spreading in a moving environment simultaneously by thermal conductivity and convection is represented by the Fourier-Kirchhoff's differential equation

$$\frac{\partial t}{\partial \tau} + w_x \frac{\partial t}{\partial x} + w_y \frac{\partial t}{\partial y} + w_z \frac{\partial t}{\partial z} = a\left(\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}\right) \tag{4}$$

More briefly, equation (4) can be written in the form

$$\frac{\partial t}{\partial \tau} + (\vec{w}, grad\ t) = a\nabla^2 t, \tag{5}$$

where the convective terms are represented by the scalar product of the velocity vectors $\vec{w}$ and the gradient of temperature *grad t*, and the conductive terms are represented by the Laplace operator $\nabla^2 t$.

The coefficient of proportionality $a$ in equations (4, 5) is called *the coefficient of thermal diffusivity*:

$$[a] = \left[\frac{\lambda}{c\rho}\right] = \left[\frac{W/(m \cdot K)}{J/(kg \cdot K) \cdot kg/m^3}\right] = \left[\frac{J/s \cdot (m \cdot K)}{J/(kg \cdot K) \cdot kg/m^3}\right] = \left[\frac{m^2}{s}\right] \tag{6}$$

The coefficient of thermal diffusivity *a* characterizes the heat-inertial properties of the body: all other things being equal, the body that has a large thermal diffusivity is heated or cooled faster [1-3].

### *Criteria for thermal similarities*

From the differential equation of convective-conductive heat transfer (4) it follows that the temperature field in a moving fluid is a function of various variables, including the velocity and density of the fluid. For practical use, equation (4) is similarly transformed taking into account the uniqueness conditions, i.e. represent as a function of the similarity criteria.

Let us first consider the similarity of boundary conditions. As was pointed out, when the liquid is turbulent, the heat is at the boundary of the flow, i.e. in the immediate vicinity of the solid wall, is represented by thermal conductivity through the boundary layer $L$ in a direction perpendicular to the direction of flow.

Consequently, according to the Fourier's law, the amount of heat passing through a boundary layer of thickness $\delta$ through the cross-sectional area $dF$ in a time $d\tau$ is

$$dQ = -\lambda \frac{\partial t}{\partial \delta} dF d\tau \qquad (7)$$

The amount of heat passing from the wall to the core of the flow is determined from the heat emission equation (8):

$$dQ = \alpha (t_w - t_l) dF d\tau \qquad (8)$$

With a steady heat exchange process, the amounts of heat passing through the boundary layer and the core of the flow are equal. Therefore, equating the expressions (7) and (8) and reducing such terms, we obtain

$$-\lambda \frac{\partial t}{\partial \delta} = \alpha (t_w - t_l) = \alpha \Delta t \qquad (9)$$

For a similar transformation of this equation, we divide its right side by the left side and discard the signs of mathematical operators. In this case, we replace $\delta$ by some defining geometric dimension $l$. Then we obtain a dimensionless complex of quantities

$$\frac{\alpha l}{\lambda} = Nu \qquad (10)$$

which is called *Nusselt's criterion*. The equality of Nusselt's criteria characterizes the similarity of heat transfer processes at the boundary between the wall and the flow of a liquid.

The division of a convective summand into a convective summand gives an expression, which is called the Peclet's criterion. The meaning of the Peclet's criterion – a measure of the ratio of the intensities of convective and conductive heat transfer in the heat carrier flow:

$$\frac{\partial(w_x t)/\partial x}{a(\partial^2 t/\partial x^2)} = \frac{(wt)/l}{a(t/l^2)} = \frac{wl}{a} = Pe \quad (11)$$

In heat-exchange processes, the difference in the density of the environment $\Delta\rho$ at various points of its volume is often a consequence of the temperature difference $\Delta t$ of this environment: $\Delta\rho = \rho\beta\Delta t$. Substitution of the expression for $\Delta\rho$ into the Archimedes criterion gives Grashof's heat criterion:

$$Gr = \frac{gl^3\beta\Delta t}{\nu^2}, \quad (12)$$

where $\beta$ – the coefficient of volumetric thermal expansion, $K^{-1}$.

Grashof's criterion is a measure of the ratio of the product of inertia forces and Archimedean lifting force to the square of the viscous friction force. The Grasgof's criterion determines the intensity of the natural thermal convection of the heat carrier in the field of gravity.

The Peclet's criterion can be represented as the product of two dimensionless complexes:

$$Pe = \frac{wl}{\nu}\cdot\frac{\nu}{a} = \frac{wl\rho}{\mu}\cdot\frac{\mu c}{\lambda} = Re\cdot Pr \quad (13)$$

The dimensionless complex

$$\frac{\nu}{a} = \frac{\mu c}{\lambda} = Pr \quad (14)$$

is called the Prandtl's criterion, which is a measure of the ratio of the viscous and thermal diffusive properties of the heat carrier.

Thus, the general relationship between the criteria that determine the intensity of the heat emission process between the flowing heat carrier and the heat exchange surface can be represented by the dependence of the Nusselt's criterion on the criteria and simplices of geometric similarity $G$:

$$Nu = f(Re, Pr, Fo, Fr, Gr, G_1, G_2 \cdots) \quad (15)$$

The form of the functions (15) is determined experimentally, and they are usually given a power-law form. Thus, for example, equation (15) for flow in a pipe of diameter $d$ and length $l$ can be represented in the form

$$Nu = CRe^m Pr^n \left(\frac{l}{d}\right)^p, \quad (16)$$

where $C$, $m$, $n$, $p$ – the values determined from the experiment.

With free motion of the fluid and for heat emission processes under natural convection, equation (15) can be represented in the form

$$Nu = CGr^m Pr^n \left(\frac{l}{d}\right)^p, \quad (17)$$

For gases $Pr \approx 1 = const$ and, therefore, the $Pr$ criterion can be excluded from the generalized equations for determining the heat emission coefficient $\alpha$ [1-3].

**Questions to control:**

1. Formulate the convective heat exchange.
2. Write the equation of heat emission.
3. Describe the criteria of thermal similarity.

**Literature**

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